Chapter 10

Angular Momentum
Problems: 5, 6, 7, 12, 29, 34, 38, 39, 49, 52, 73
Think about: 31, 32, 40, 66, 79

5 • **[SSM]** A particle travels in a circular path and point *P* is at the center of the circle. (*a*) If the particle's linear momentum $\vec{p}$ is doubled without changing the radius of the circle, how is the magnitude of its angular momentum about *P* affected? (*b*) If the radius of the circle is doubled but the speed of the particle is unchanged, how is the magnitude of its angular momentum about *P* affected?

Determine the Concept $\vec{L}$ and $\vec{p}$ are related according to $\vec{L} = \vec{r} \times \vec{p}$.

(*a*) Because $\vec{L}$ is directly proportional to $\vec{p}$, *L* is doubled.

(*b*) Because $\vec{L}$ is directly proportional to $\vec{r}$, *L* is doubled.

6 • A particle moves along a straight line at constant speed. How does its angular momentum about any fixed point vary with time?

Determine the Concept We can determine how the angular momentum of the particle about any fixed point varies with time by examining the derivative of the cross product of $\vec{r}$ and $\vec{p}$.

The angular momentum of the particle is given by:

$$\vec{L} = \vec{r} \times \vec{p}$$

Differentiate $\vec{L}$ with respect to time to obtain:

$$\frac{d\vec{L}}{dt} = \left(\vec{r} \times \frac{d\vec{p}}{dt}\right) + \left(\frac{d\vec{r}}{dt} \times \vec{p}\right) \quad (1)$$

Because $\vec{p} = m\vec{v}$, $\frac{d\vec{p}}{dt} = \vec{F}_{\text{net}}$, and $\frac{d\vec{r}}{dt} = \vec{v}$:

$$\frac{d\vec{L}}{dt} = \left(\vec{r} \times \vec{F}_{\text{net}}\right) + \left(\vec{v} \times \vec{p}\right)$$

Because the particle moves along a straight line at constant speed:

$$F_{\text{net}} = 0 \Rightarrow \vec{r} \times \vec{F}_{\text{net}} = 0$$

Because $\vec{v}$ and $\vec{p}\left(= m\vec{v}\right)$ are parallel:

$$\vec{v} \times \vec{p} = 0$$

Substitute in equation (1) to obtain:

$$\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} \text{ does not change in time.}$$

7 •• True or false: If the net torque on a rotating system is zero, the angular velocity of the system cannot change. If your answer is false, give an example of such a situation.

False. The net torque acting on a rotating system equals the change in the system's angular momentum; that is, $\tau_{\text{net}} = dL/dt$ where $L = I\omega$. Hence, if τ_{net} is zero, all we can say for sure is that the angular momentum (the product of I and ω) is constant. If I changes, so must ω. An example is a high diver going from a tucked to a layout position.

12 •• Explain why a helicopter with just one main rotor has a second smaller rotor mounted on a horizontal axis at the rear as in Figure 10-40. Describe the resultant motion of the helicopter if this rear rotor fails during flight.

Determine the Concept The purpose of the second smaller rotor is to prevent the body of the helicopter from rotating. If the rear rotor fails, the body of the helicopter will tend to rotate on the main axis due to angular momentum being conserved.

29 • Find $\vec{A}\times\vec{B}$ for the following choices: (*a*) $\vec{A} = 4\hat{\boldsymbol{i}}$ and $\vec{B} = 6\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{j}}$, (*b*) $\vec{A} = 4\hat{\boldsymbol{i}}$ and $\vec{B} = 6\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{k}}$, and (*c*) $\vec{A} = 2\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}}$ and $\vec{B} = 3\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}}$.

Picture the Problem We can use the definitions of the cross products of the unit vectors $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$, and $\hat{\boldsymbol{k}}$ to evaluate $\vec{A}\times\vec{B}$ in each case.

(*a*) Evaluate $\vec{A}\times\vec{B}$ for $\vec{A} = 4\hat{\boldsymbol{i}}$ and $\vec{B} = 6\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{j}}$:

$$\begin{aligned}\vec{A}\times\vec{B} &= 4\hat{\boldsymbol{i}}\times\left(6\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{j}}\right)\\ &= 24\left(\hat{\boldsymbol{i}}\times\hat{\boldsymbol{i}}\right) + 24\left(\hat{\boldsymbol{i}}\times\hat{\boldsymbol{j}}\right)\\ &= 24(0) + 24\hat{\boldsymbol{k}}\\ &= \boxed{24\hat{\boldsymbol{k}}}\end{aligned}$$

(*b*) Evaluate $\vec{A}\times\vec{B}$ for $\vec{A} = 4\hat{\boldsymbol{i}}$ and $\vec{B} = 6\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{k}}$:

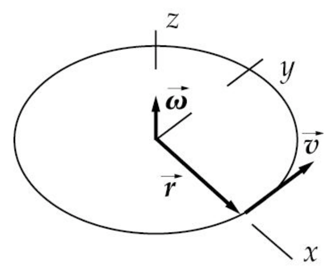
$$\begin{aligned}\vec{A}\times\vec{B} &= 4\hat{\boldsymbol{i}}\times\left(6\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{k}}\right)\\ &= 24\left(\hat{\boldsymbol{i}}\times\hat{\boldsymbol{i}}\right) + 24\left(\hat{\boldsymbol{i}}\times\hat{\boldsymbol{k}}\right)\\ &= 24(0) + 24\left(-\hat{\boldsymbol{j}}\right)\\ &= \boxed{-24\hat{\boldsymbol{j}}}\end{aligned}$$

(*c*) Evaluate $\vec{A}\times\vec{B}$ for $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=3\hat{i}+2\hat{j}$:

$$\begin{aligned}\vec{A}\times\vec{B} &= \left(2\hat{i}+3\hat{j}\right)\times\left(3\hat{i}+2\hat{j}\right)\\ &= 6\left(\hat{i}\times\hat{i}\right)+4\left(\hat{i}\times\hat{j}\right)+9\left(\hat{j}\times\hat{i}\right)\\ &\quad+6\left(\hat{j}\times\hat{j}\right)\\ &= 6(0)+4\left(\hat{k}\right)+9\left(-\hat{k}\right)+6(0)\\ &= \boxed{-5\hat{k}}\end{aligned}$$

31 •• A particle moves in a circle that is centered at the origin. The particle has position $\vec{r}$ and angular velocity $\vec{\omega}$. (*a*) Show that its velocity is given by $\vec{v}=\vec{\omega}\times\vec{r}$. (*b*) Show that its centripetal acceleration is given by $\vec{a}_c=\vec{\omega}\times\vec{v}=\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)$.

Picture the Problem Let $\vec{r}$ be in the *xy* plane and point in the +*x* direction. Then $\vec{\omega}$ points in the +*z* direction. We can establish the results called for in this problem by forming the appropriate cross products and by differentiating $\vec{v}$.



(*a*) Express $\vec{\omega}$ using unit vector notation:

$$\vec{\omega}=\omega\hat{\mathbf{k}}$$

Express $\vec{r}$ using unit vector notation:

$$\vec{r}=r\hat{i}$$

Form the cross product of $\vec{\omega}$ and $\vec{r}$:

$$\vec{\omega}\times\vec{\mathbf{r}}=\omega\hat{\mathbf{k}}\times r\hat{\mathbf{i}}=r\omega\left(\hat{\mathbf{k}}\times\hat{\mathbf{i}}\right)=r\omega\,\hat{\mathbf{j}}$$
$$=v\hat{\mathbf{j}}$$

and $\vec{\mathbf{v}}=\boxed{\vec{\omega}\times\vec{\mathbf{r}}}$

(*b*) Differentiate $\vec{v}$ with respect to *t* to express $\vec{a}$:

$$\vec{\mathbf{a}}=\frac{d\vec{\mathbf{v}}}{dt}=\frac{d}{dt}\left(\vec{\omega}\times\vec{\mathbf{r}}\right)=\frac{d\vec{\omega}}{dt}\times\vec{\mathbf{r}}+\vec{\omega}\times\frac{d\vec{\mathbf{r}}}{dt}=\frac{d\vec{\omega}}{dt}\times\vec{\mathbf{r}}+\vec{\omega}\times\vec{\mathbf{v}}=\vec{\mathbf{a}}_t+\vec{\omega}\times\left(\vec{\omega}\times\vec{\mathbf{r}}\right)$$
$$=\vec{\mathbf{a}}_t+\vec{\mathbf{a}}_c$$

where $\vec{\mathbf{a}}_c=\boxed{\vec{\omega}\times\left(\vec{\omega}\times\vec{\mathbf{r}}\right)}$ and $\vec{a}_t$ and $\vec{a}_c$ are the tangential and centripetal accelerations, respectively.

32 •• You are given three vectors and their components in the form: $\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, $\vec{B} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$, and $\vec{C} = c_x\hat{i} + c_y\hat{j} + c_z\hat{k}$. Show that the following equalities hold: $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{C}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{C}\times\vec{A})$

Picture the Problem We can establish these equalities by carrying out the details of the cross- and dot-products and comparing the results of these operations.

Evaluate the cross product of $\vec{B}$ and $\vec{C}$ to obtain:

$$\vec{B}\times\vec{C} = (b_yc_z - b_zc_y)\hat{i} + (b_zc_x - b_xc_z)\hat{j} + (b_xc_y - b_yc_x)\hat{k}$$

Form the dot product of $\vec{A}$ with $\vec{B}\times\vec{C}$ to obtain:

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = a_xb_yc_z - a_xb_zc_y + a_yb_zc_x - a_yb_xc_z + a_zb_xc_y - a_zb_yc_x \quad (1)$$

Evaluate the cross product of $\vec{A}$ and $\vec{B}$ to obtain:

$$\vec{A}\times\vec{B} = (a_yb_z - a_zb_y)\hat{i} + (a_zb_x - a_xb_z)\hat{j} + (a_xb_y - a_yb_x)\hat{k}$$

Form the dot product of $\vec{C}$ with $\vec{A}\times\vec{B}$ to obtain:

$$\vec{C}\cdot(A\times\vec{B}) = c_xa_yb_z - c_xa_zb_y + c_ya_zb_x - c_ya_xb_z + c_za_xb_y - c_za_yb_x \quad (2)$$

Evaluate the cross product of $\vec{C}$ and $\vec{A}$ to obtain:

$$\vec{C}\times\vec{A} = (c_ya_z - a_za_y)\hat{i} + (c_za_x - a_xa_z)\hat{j} + (c_xa_y - a_ya_x)\hat{k}$$

Form the dot product of $\vec{B}$ with $\vec{C}\times\vec{A}$ to obtain:

$$\vec{B}\cdot(\vec{C}\times\vec{A}) = b_xc_ya_z - b_xc_za_y + b_yc_za_x - b_yc_xa_z + b_zc_xa_y - b_zc_ya_x \quad (3)$$

The equality of equations (1), (2), and (3) establishes the equalities.

34 •• If $\vec{A} = 4\hat{i}$, $B_z = 0$, $|\vec{B}| = 5$, and $\vec{A}\times\vec{B} = 12\hat{k}$, determine $\vec{B}$.

Picture the Problem Because $B_z = 0$, we can express $\vec{B}$ as $\vec{B} = B_x\hat{i} + B_y\hat{j}$ and form its cross product with $\vec{A}$ to determine B_x and B_y.

Express $\vec{B}$ in terms of its components: $\quad \vec{B} = B_x\hat{i} + B_y\hat{j} \quad (1)$

Express $\vec{A}\times\vec{B}$:

$$\vec{A}\times\vec{B} = 4\hat{i}\times\left(B_x\hat{i} + B_y\hat{j}\right) = 4B_y\hat{k} = 12\hat{k}$$

Solving for B_y yields:

$$B_y = 3$$

Relate B to B_x and B_y:

$$B^2 = B_x^2 + B_y^2$$

Solve for and evaluate B_x:

$$B_x = \sqrt{B^2 - B_y^2} = \sqrt{5^2 - 3^2} = 4$$

Substitute for B_x and B_y in equation (1) to obtain:

$$\vec{B} = \boxed{4\hat{i} + 3\hat{j}}$$

38 • You observe a 2.0-kg particle moving at a constant speed of 3.5 m/s in a clockwise direction around a circle of radius 4.0 m. (*a*) What is its angular momentum (including direction) about the center of the circle? (*b*) What is its moment of inertia about an axis through the center of the circle and perpendicular to the plane of the motion? (*c*) What is the angular velocity of the particle?

Picture the Problem The angular momentum of the particle is $\vec{L} = \vec{r}\times\vec{p}$ where $\vec{r}$ is the vector locating the particle relative to the reference point and $\vec{p}$ is the particle's linear momentum.

(*a*) The magnitude of the particle's angular momentum is given by:

$$L = rp\sin\phi = rmv\sin\phi = mv\left(r\sin\phi\right)$$

Substitute numerical values and evaluate the magnitude of L:

$$\boldsymbol{L} = (2.0\,\text{kg})(3.5\,\text{m/s})(4.0\,\text{m}) = 28\,\text{kg}\cdot\text{m}^2/\text{s}$$

Use a right-hand rule to establish the direction of $\vec{L}$:

$$\boldsymbol{L} = \boxed{28\,\text{kg}\cdot\text{m}^2/\text{s, away from you}}$$

(*b*) Treat the 2.0-kg particle as a point particle to obtain:

$$I = mr^2$$

Substitute numerical values and evaluate I:

$$\boldsymbol{I} = (2.0\,\text{kg})(4.0\,\text{m})^2 = \boxed{32\,\text{kg}\cdot\text{m}^2}$$

(*c*) Because $L = I\omega$, the angular speed of the particle is the ratio of its angular momentum and its moment of inertia:

$$\omega = \frac{L}{I} \text{ or } \omega = \frac{v}{r}$$

Substitute numerical values and evaluate ω:

$$\omega = \frac{28\,\text{kg}\cdot\text{m}^2/\text{s}}{32\,\text{kg}\cdot\text{m}^2} = \boxed{0.88\,\text{rad/s}^2}$$

39 •• (*a*) A particle moving at constant velocity has zero angular momentum about a particular point. Use the definition of angular momentum to show that under this condition the particle is moving either directly toward or directly away from the point. (*b*) You are a right-handed batter and let a waist-high fastball go past you without swinging. What is the direction of its angular momentum relative to your navel? (Assume the ball travels in a straight horizontal line as it passes you.)

Picture the Problem $\vec{L}$ and $\vec{p}$ are related according to $\vec{L} = \vec{r} \times \vec{p}$. If $\vec{L} = 0$, then examination of the magnitude of $\vec{r} \times \vec{p}$ will allow us to conclude that $\sin\phi = 0$ and that the particle is moving either directly toward the point, directly away from the point, or through the point.

(*a*) Because $\vec{L} = 0$:

$$\vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = m\vec{r} \times \vec{v} = 0$$

or

$$\vec{r} \times \vec{v} = 0$$

Express the magnitude of $\vec{r} \times \vec{v}$:

$$\left|\vec{r} \times \vec{v}\right| = rv\sin\phi = 0$$

Because neither r nor v is zero:

$$\sin\phi = 0$$

where ϕ is the angle between $\vec{r}$ and $\vec{v}$.

Solving for ϕ yields:

$$\phi = \sin^{-1}(0) = \boxed{0° \text{ or } 180°}$$

(*b*) Use the right-hand rule to establish that the ball's angular momentum is downward.

40 •• A particle that has a mass m is traveling with a constant velocity $\vec{v}$ along a straight line that is a distance b from the origin O (Figure 10-44). Let dA be the area swept out by the position vector from O to the particle during a time interval dt. Show that dA/dt is constant and is equal to $L/2m$, where L is the magnitude of the angular momentum of the particle about the origin.

Picture the Problem We can use the formula for the area of a triangle to find the area swept out at $t = t_1$, add this area to the area swept out in time dt, and then differentiate this expression with respect to time to obtain the given expression for dA/dt.

Express the area swept out at $t = t_1$: $A_1 = \frac{1}{2}br_1\cos\theta_1 = \frac{1}{2}bx_1$

where θ_1 is the angle between $\vec{r}_1$ and $\vec{v}$ and x_1 is the component of $\vec{r}_1$ in the direction of $\vec{v}$.

The area swept out at $t = t_1 + dt$ is: $\boldsymbol{A = A_1 + dA}$

Substitute for A_1 to obtain: $\boldsymbol{A = A_1 + dA = \frac{1}{2}b(x_1 + dx)}$

Because $dx = vdt$: $\boldsymbol{A = \frac{1}{2}b(x_1 + vdt)}$

Differentiate A with respect to t to obtain:

$$\frac{dA}{dt} = \frac{1}{2}b\frac{dx}{dt} = \frac{1}{2}bv = \text{constant}$$

Because $r\sin\theta = b$:

$$\frac{1}{2}bv = \frac{1}{2}(r\sin\theta)v = \frac{1}{2m}(rp\sin\theta) = \boxed{\frac{L}{2m}}$$

49 •• **[SSM]** You stand on a frictionless platform that is rotating at an angular speed of 1.5 rev/s. Your arms are outstretched, and you hold a heavy weight in each hand. The moment of inertia of you, the extended weights, and the platform is 6.0 kg·m^2. When you pull the weights in toward your body, the moment of inertia decreases to 1.8 kg·m^2. (*a*) What is the resulting angular speed of the platform? (*b*) What is the change in kinetic energy of the system? (*c*) Where did this increase in energy come from?

Picture the Problem Let the system consist of you, the extended weights, and the platform. Because the net external torque acting on this system is zero, its angular momentum remains constant during the pulling in of the weights.

(*a*) Using conservation of angular momentum, relate the initial and final angular speeds of the system to its initial and final moments of inertia:

$$\boldsymbol{I}_i\boldsymbol{\omega}_i - \boldsymbol{I}_f\boldsymbol{\omega}_f = 0 \Rightarrow \omega_f = \frac{I_i}{I_f}\omega_i$$

Substitute numerical values and evaluate ω_f:

$$\boldsymbol{\omega}_f = \frac{6.0\,\text{kg}\cdot\text{m}^2}{1.8\,\text{kg}\cdot\text{m}^2}(1.5\,\text{rev/s}) = \boxed{5.0\,\text{rev/s}}$$

(*b*) Express the change in the kinetic energy of the system:

$$\Delta K = K_f - K_i = \tfrac{1}{2} I_f \omega_f^2 - \tfrac{1}{2} I_i \omega_i^2$$

Substitute numerical values and evaluate ΔK:

$$\Delta K = \tfrac{1}{2}\left(1.8\,\text{kg}\cdot\text{m}^2\right)\left(5.0\frac{\text{rev}}{\text{s}}\times\frac{2?\,\text{rad}}{\text{rev}}\right)^2 - \tfrac{1}{2}\left(6.0\,\text{kg}\cdot\text{m}^2\right)\left(1.5\frac{\text{rev}}{\text{s}}\times\frac{2?\,\text{rad}}{\text{rev}}\right)^2$$
$$= \boxed{0.62\,\text{kJ}}$$

(*c*) Because no external agent does work on the system, the energy comes from your internal energy.

52 •• Two disks of identical mass but different radii (r and $2r$) are spinning on frictionless bearings at the same angular speed ω_0 but in opposite directions (Figure 10-49). The two disks are brought slowly together. The resulting frictional force between the surfaces eventually brings them to a common angular velocity. (*a*) What is the magnitude of that final angular velocity in terms of ω_0? (*b*) What is the change in rotational kinetic energy of the system? Explain.

Picture the Problem The net external torque acting on this system is zero and so we know that angular momentum is conserved as these disks are brought together. Let the numeral 1 refer to the disk to the left and the numeral 2 to the disk to the right. Let the angular momentum of the disk with the larger radius be positive.

(*a*) Using conservation of angular momentum, relate the initial angular speeds of the disks to their common final speed and to their moments of inertia:

$$I_i\omega_i = I_f\omega_f$$
or
$$I_1\omega_0 - I_2\omega_0 = \left(I_1 + I_2\right)\omega_f$$

Solving for ω_f yields:

$$\omega_f = \frac{I_1 - I_2}{I_1 + I_2}\omega_0 \qquad (1)$$

Express I_1 and I_2:

$$I_1 = \tfrac{1}{2}m(2r)^2 = 2mr^2$$
and
$$I_2 = \tfrac{1}{2}mr^2$$

Substitute for I_1 and I_2 in equation (1) and simplify to obtain:

$$\omega_f = \frac{2mr^2 - \tfrac{1}{2}mr^2}{2mr^2 + \tfrac{1}{2}mr^2}\omega_0 = \boxed{\tfrac{3}{5}\omega_0}$$

(*b*) The change in kinetic energy of the system is given by:

$$\Delta K = K_f - K_i \qquad (2)$$

The initial kinetic energy of the system is the sum of the kinetic energies of the two disks:

$$\begin{aligned} \boldsymbol{K}_i &= \boldsymbol{K}_1 + \boldsymbol{K}_2 \\ &= \tfrac{1}{2}\boldsymbol{I}_1\boldsymbol{\omega}_0^2 + \tfrac{1}{2}\boldsymbol{I}_2\boldsymbol{\omega}_0^2 \\ &= \tfrac{1}{2}(\boldsymbol{I}_1 + \boldsymbol{I}_2)\boldsymbol{\omega}_0^2 \end{aligned}$$

Substituting for K_f and K_i in equation (2) yields:

$$\Delta K = \tfrac{1}{2}(I_1 + I_2)\omega_f^2 - \tfrac{1}{2}(I_1 + I_2)\omega_0^2$$

Substitute for ω_f from part (*a*) and simplify to obtain:

$$\begin{aligned} \Delta K &= \tfrac{1}{2}(I_1 + I_2)\left(\tfrac{3}{5}\omega_0\right)^2 - \tfrac{1}{2}(I_1 + I_2)\omega_0^2 \\ &= -\tfrac{16}{25}\left[\tfrac{1}{2}(I_1 + I_2)\omega_0^2\right] \end{aligned}$$

Noting that the quantity in brackets is K_i, substitute to obtain:

$$\Delta K = \boxed{-\tfrac{16}{25}K_i}$$

The frictional force between the surfaces is responsible for some of the initial kinetic energy being converted to thermal energy as the two disks come together.

66 •• A projectile of mass m_p is traveling at a constant velocity $\vec{v}_0$ toward a stationary disk of mass M and radius R that is free to rotate about its axis O (Figure 10-54). Before impact, the projectile is traveling along a line displaced a distance b below the axis. The projectile strikes the disk and sticks to point B. Model the projectile as a point mass. (*a*) Before impact, what is the total angular momentum L_0 of the disk-projectile system about the axis? Answer the following questions in terms of the symbols given at the start of this problem. (*b*) What is the angular speed ω of the disk-projectile system just after the impact? (*c*) What is the kinetic energy of the disk-projectile system after impact? (*d*) How much mechanical energy is lost in this collision?

Picture the Problem Because the net external torque acting on the system is zero, angular momentum is conserved in this perfectly inelastic collision.

(*a*) Use its definition to express the total angular momentum of the disk and projectile just before impact:

$$L_0 = \boxed{m_p v_0 b}$$

(*b*) Use conservation of angular momentum to relate the angular

$$L_0 = L = I\omega \Rightarrow \omega = \frac{L_0}{I}$$

momenta just before and just after the collision:

The moment of inertia of the disk-projectile after the impact is:

$$I = \tfrac{1}{2}MR^2 + m_\mathrm{p}R^2 = \tfrac{1}{2}(M + 2m_\mathrm{p})R^2$$

Substitute for I in the expression for ω to obtain:

$$\boxed{\omega = \frac{2m_\mathrm{p}v_0 b}{(M + 2m_\mathrm{p})R^2}}$$

(c) Express the kinetic energy of the system after impact in terms of its angular momentum:

$$K_\mathrm{f} = \frac{L^2}{2I} = \frac{(m_\mathrm{p}v_0 b)^2}{2\left[\tfrac{1}{2}(M + 2m_\mathrm{p})R^2\right]} = \boxed{\frac{(m_\mathrm{p}v_0 b)^2}{(M + 2m_\mathrm{p})R^2}}$$

(d) Express the difference between the initial and final kinetic energies, substitute, and simplify to obtain:

$$\Delta E = K_\mathrm{i} - K_\mathrm{f}$$

$$= \tfrac{1}{2}m_\mathrm{p}v_0^2 - \frac{(m_\mathrm{p}v_0 b)^2}{(M + 2m_\mathrm{p})R^2}$$

$$= \boxed{\tfrac{1}{2}m_\mathrm{p}v_0^2\left[1 - \frac{2m_\mathrm{p}b^2}{(M + 2m_\mathrm{p})R^2}\right]}$$

73 •• Two ice skaters, whose masses are 55 kg and 85 kg, hold hands and rotate about a vertical axes that passes between them, making one revolution in 2.5 s. Their centers of mass are separated by 1.7 m and their center of mass is stationary. Model each skater as a point particle and find (a) the angular momentum of the system about their center of mass and (b) the total kinetic energy of the system.

Picture the Problem The ice skaters rotate about their center of mass; a point we can locate using its definition. Knowing the location of the center of mass we can determine their moment of inertia with respect to an axis through this point. The angular momentum of the system is then given by $L = I_\mathrm{cm}\omega$ and its kinetic energy can be found from $K = L^2/(2I_\mathrm{cm})$.

Express the angular momentum of the system about the center of mass of the skaters:

$$L = I_\mathrm{cm}\omega$$

Using its definition, locate the center of mass, relative to the 85-kg skater, of the system:

$$x_{cm} = \frac{(55\,\text{kg})(1.7\,\text{m}) + (85\,\text{kg})(0)}{55\,\text{kg} + 85\,\text{kg}} = 0.668\,\text{m}$$

Calculate I_{cm}:

$$I_{cm} = (55\,\text{kg})(1.7\,\text{m} - 0.668\,\text{m})^2 + (85\,\text{kg})(0.668\,\text{m})^2 = 96.5\,\text{kg}\cdot\text{m}^2$$

Substitute to determine L:

$$L = (96.5\,\text{kg}\cdot\text{m}^2)\left(\frac{1\,\text{rev}}{2.5\,\text{s}} \times \frac{2\partial\,\text{rad}}{\text{rev}}\right) = 243\,\text{J}\cdot\text{s} = \boxed{0.24\,\text{kJ}\cdot\text{s}}$$

(*b*) Relate the total kinetic energy of the system to its angular momentum and evaluate K:

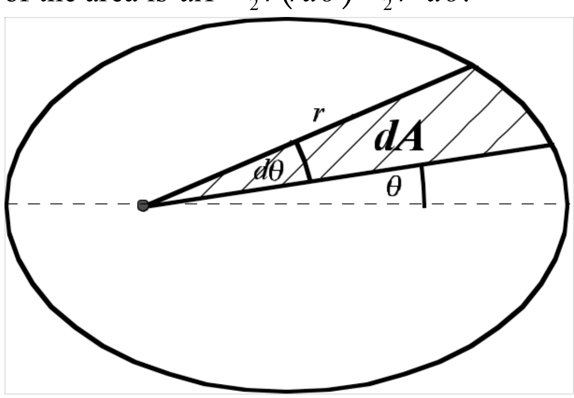
$$K = \frac{L^2}{2I_{cm}}$$

Substitute numerical values and evaluate K:

$$\boldsymbol{K} = \frac{(243\,\text{J}\cdot\text{s})^2}{2(96.5\,\text{kg}\cdot\text{m}^2)} = \boxed{0.31\,\text{kJ}}$$

79 •• **[SSM]** Kepler's second law states: *The line from the center of the Sun to the center of a planet sweeps out equal areas in equal times.* Show that this law follows directly from the law of conservation of angular momentum and the fact that the force of gravitational attraction between a planet and the Sun acts along the line joining the centers of the two celestial objects.

Picture the Problem The pictorial representation shows an elliptical orbit. The triangular element of the area is $dA = \frac{1}{2}r(rd\theta) = \frac{1}{2}r^2 d\theta$.



Differentiate dA with respect to t to obtain:

$$\frac{dA}{dt} = \tfrac{1}{2}r^2\frac{d\theta}{dt} = \tfrac{1}{2}r^2\omega \qquad (1)$$

Because the gravitational force acts along the line joining the two objects, $\tau = 0$. Hence:

$$L = mr^2\omega = \text{constant} \qquad (2)$$

Eliminate $r^2\omega$ between equations (1) and (2) to obtain:

$$\frac{dA}{dt} = \boxed{\frac{L}{2m} = \text{constant}}$$